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PNN - COCOSO FOR MULTI ATTRIBUTE GROUP DECISION MAKING

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Abstract: Decision making in real-world situations is highly challenging during evaluation of a set of alternatives with conflicting criteria. Although there exists a vast body of literature covering different Multi-Attribute Decision Making (MADM) frameworks under single or group decision making (MAGDM), most of them are limited to fuzzy or neutrosophic environments capturing limited uncertainty. To address these limitations, this paper proposes an extension of the Combined Compromise Solution (COCOSO) strategy for group decision making within the framework of Pentapartitioned Neutrosophic Numbers (PNNs), which we name as PNN - COCOSO strategy. Unlike classical fuzzy or single-valued neutrosophic approaches, pentapartitioned neutrosophic numbers allow decision makers to distinguish among truth, falsity, contradiction, and ignorance, providing a comprehensive picture of uncertainty in decision making.

Furthermore, this paper utilizes COCOSO's mechanism of multiple compromise measures through a distinguished parameter Ω, combining additive and multiplicative aggregation simultaneously to enhance ranking stability.To illustrate the developed strategy, a benchmark problem of group decision making is selected from literature and analyzed for any alteration of ranking. Additionally, a secondary illustration is also provided to present the case of single decision maker. A comprehensive sensitivity analysis is performed to ascertain the stability of rankings across the ranges of the distinguished parameter $\Omega \in [0.1, 1.0]$, enhancing the robustness of the proposed method in decision making.

Keywords and Phrases: COCOSO method, Multi-Attribute Group Decision Making (MAGDM), Pentapartitioned Neutrosophic Number (PNN), Appraisal Score, Sensitivity Analysis.

2020 Mathematics Subject Classification: 08A99, 03E99, 03B52.

1. Introduction

Uncertainty modeling in mathematics was initiated with the introduction of membership functions in fuzzy set theory [30] to capture inconsistencies in real-world data sets. This foundational concept was further extended by incorporating non-membership functions alongside it in the intuitionistic theory of fuzzy sets [1]. However, to address incomplete and indeterminate information more explicitly, a new component of indeterminacy was introduced by Smarandache [25] in the theory of Neutrosophic Sets (NSs). To enhance practical applicability, NS was refined into Single-Valued NS (SVNS) [28]. Following this line of development, the SVNS model was extended to Quadripartitioned SVNS (QSVNS) [3], based on the principles of n-valued logic [26] and four-valued logic [2]. The concept of Pentapartitioned NS (PNS) [15] was developed by extending this line, where the indeterminacy component is further decomposed into three independent components of "contradiction", "ignorance" and "unknown" which, when aligned with "truth" and "falsity" generates a five-tuple structure. The corresponding elemental representation of PNS emerged as the Pentapartitioned Neutrosophic Number (PNN) [17], which serves as a significant mathematical tool for uncertainty modeling.

In the domain of Multi-Attribute Decision Making (MADM) [9], a wide spectrum of methods has been proposed to select the most optimized option from a heterogeneous combination of conflicting criteria. Classical approaches such as TOPSIS [6], TODIM [16], MABAC [18], VIKOR [19], COCOSO [29] and AHP [23] have been successfully adapted to fuzzy and neutrosophic environments. Additionally, a Neutrosophic BWM-TOPSIS strategy [21] was developed combining the Best-Worst Method (BWM) for calculating criteria weights with the TOPSIS method for ranking alternatives.

To apply these MADM approaches within PNN contexts, several aggregation operators, such as weighted arithmetic [20] and geometric operators [17], were formulated to facilitate practical applicability in decision-making. Through the development of these aggregation operators, classical decision-making approaches like SVPNN-ARAS [20], GRA [5] were extended to PNN settings. Other notable works in the PNN environment include the introduction of various distance measures and their integration into MADM frameworks. Significant contributions involve weighted hyperbolic tangent similarity measures [14] to capture complex relationships between variables, and dice similarity measures [24] that evaluate alternatives by analyzing the overlap between truth, falsity, contradiction, ignorance, and unknown components. Furthermore, hyperbolic sine similarity [4], which provides a trigonometry-based distance measure, and exponential similarity measures [8], which define distance based on exponential functions, have been successfully integrated into MADM frameworks while maintaining PNN structures.

Recent advances in the field continue to expand the flexibility of these mathematical structures through novel aggregation operators and integrated decision-making methodologies. Aczel-Alsina aggregation operators [11] were developed over Interval-Valued Complex Single-Valued Neutrosophic (IVCSVN) settings and integrated into an MADM framework. Similarly, Prioritized Muirhead Mean (PMM) operators [12] involving complex neutrosophic sets were used to investigate interrelationships between partitioned arguments in these settings. Other innovations include the integration of Dombi operators with Bonferroni mean operators [13], which combine multi-valued neutrosophic uncertain linguistic sets with complex fuzzy sets.

Simultaneously, integrated MADM approaches are increasingly being applied to strategic sector challenges. The combined use of LODECI and CORASO methods [27] has been introduced to handle the complexities of multiple criteria and subcriteria for rigorous supplier selection in the food sector. Hybrid methodologies, such as SWARA-MABAC [18], were formulated to determine criteria weights and execute decision making using fuzzy triangular numbers. Another recent study in 2026 indicates a combined approach [7] involving DELPHI, SWARA, and Fuzzy WASPAS for decision making in uncertain environments. Furthermore, integrating Pythagorean Neutrosophic TOPSIS and VIKOR [19] with flexible novel distance metrics and an indeterminacy quantifier was introduced for the evaluation of digital suppliers. Other hybrid approaches [10] combine Fuzzy DEMATEL and BWM with COCOSO, MOORA, and TOPSIS to enhance transparency in supplier selection within fuzzy environments.

Although the aforementioned methods and aggregation operators were strategized for fuzzy or neutrosophic frameworks handling limited uncertainties, notable advancements have also been observed in the field of PNN. A recent study of 2025 introduced Pentapartitioned Neutrosophic Dombi Weighted (Arithmetic and Geometric) Aggregation Operators (PNDWAA and PNDWGA) [22]. These operators leverage the adaptability of Dombi T-norm and T-conorm operations to achieve precise data aggregation under a PNN environment, but they currently lack an established MADM framework.

Research Gap and Motivation

Despite these advancements, most MADM frameworks remain primarily focused on neutrosophic or complex neutrosophic settings, which only project a combined measure of indeterminacy. Although they involve different aggregation operators [11], [12], [13], they lack granular decomposition and the precise measurement of uncertainty due to the absence of a PNN environment. Furthermore, some of the supply chain models cited above concentrate more on criteria and sub-criteria selection rather than providing a comprehensive MADM framework for uncertain environments.

While there have been recent developments in PNN settings, such as SVPNN-ARAS [20], its reliance on a single aggregation mechanism and a non-normalized score function may not yield reliable results. On the other hand, the incorporation of Dombi operators [22] in PNN has produced individual aggregation mechanisms, but it still lacks combined aggregation and a formalized MADM framework.

To mitigate these limitations and bridge this gap, this paper proposes a novel PNN-COCOSO strategy. The primary novelty of this method lies in its use of a normalized score function, the integration of multiple aggregation mechanisms (such as arithmetic and geometric), and its adherence to a conventional MADM framework. Moreover, by grounding the approach in the PNN environment, it enables granular decomposition and an appropriate measurement of uncertainty through its five-component structure.

2. Preliminaries

Definition 1. (**Pentapartitioned Neutrosophic Set (PNS)**)
A Pentapartitioned Neutrosophic Set (PNS) [15] $\mathcal{S}(x)$ *defined over a universe of discourse* X *is denoted as*

$$\mathcal{S}(x) = \langle x \in X : \tau(x), \chi(x), \iota(x), \nu(x), \phi(x) \rangle,$$

where, $\tau(x), \chi(x), \iota(x), \nu(x), \phi(x)$ *represent the truth Membership Function (MF), contradiction MF, ignorance MF, unknown MF and falsity MF respectively.*

Each of these components satisfies the conditions

$$\tau(x), \chi(x), \iota(x), \nu(x), \phi(x) \in [0,1] \forall x \in X \text{ and } 0 \leq \tau(x)+\chi(x)+\iota(x)+\nu(x)+\phi(x) \leq 5.$$

Definition 2. (**Operations of PNS**) [15]
Let us consider two PNS $\mathcal{S}_1(x), \mathcal{S}_2(x)$ as

$$\mathcal{S}_1(x) = \langle x \in X : \tau_1(x), \chi_1(x), \iota_1(x), \nu_1(x), \phi_1(x) \rangle,$$

$$\mathcal{S}_2(x) = \langle x \in X : \tau_2(x), \chi_2(x), \iota_2(x), \nu_2(x), \phi_2(x) \rangle,$$

They satisfy the set operations as defined below.

- ***Subset*** - $\mathcal{S}_1(x) \subseteq \mathcal{S}_2(x)$ *iff*

$$\tau_1(x) \leq \tau_2(x), \chi_1(x) \leq \chi_2(x), \iota_1(x) \geq \iota_2(x), \nu_1(x) \geq \nu_2(x), \phi_1(x) \geq \phi_2(x)$$

- ***Union*** - $\mathcal{S}_1(x) \cup \mathcal{S}_2(x)$
$= \langle max(\tau_1(x), \tau_2(x)), max(\chi_1(x), \chi_2(x)), min(\iota_1(x), \iota_2(x)),$
$min(\nu_1(x), \nu_2(x)), min(\phi_1(x), \phi_2(x)) \rangle$

- ***Intersection*** - $\mathcal{S}_1(x) \cap \mathcal{S}_2(x)$
$= \langle min(\tau_1(x), \tau_2(x)), min(\chi_1(x), \chi_2(x)), max(\iota_1(x), \iota_2(x)),$
$max(\nu_1(x), \nu_2(x)), max(\phi_1(x), \phi_2(x)) \rangle$

- ***Complement*** -

$$\mathcal{S}_1^C(x) = \langle x \in X : \phi_1(x), \nu_1(x), 1 - \iota_1(x), \chi_1(x), \tau_1(x) \rangle$$

Definition 3. (**Pentapartitioned Neutrosophic Number (PNN)**) [17]
Let $\mathcal{S}(x)$ be a PNS and $\tilde{A} \in \mathcal{S}$, then $\tilde{A}$ is denoted as

$$\tilde{A} = \langle \tau, \chi, \iota, \nu, \phi \rangle,$$

where, $\tau, \chi, \iota, \nu, \phi$ represent the truth Membership Degree (MD), contradiction MD, ignorance MD, unknown MD and falsity MD respectively.

Each of these components satisfies the condition

$$\tau, \chi, \iota, \nu, \phi \in [0, 1] \quad and \quad 0 \leq \tau + \chi + \iota + \nu + \phi \leq 5.$$

Such an element ($\tilde{A}$) is referred to as a Pentapartitioned Neutrosophic Number (PNN) .
PNN represents the elemental form of a PNS.
Let us now consider two PNNs as

$$\tilde{A}_1 = \langle \tau_1, \chi_1, \iota_1, \nu_1, \phi_1 \rangle \quad \text{and} \quad \tilde{A}_2 = \langle \tau_2, \chi_2, \iota_2, \nu_2, \phi_2 \rangle.$$

They satisfy the arithmetic operations of PNN addition ($\oplus$), multiplication ($\otimes$), scalar multiplication (*) and power operations as defined below.

Definition 4. (**Operations of PNN**) [17]

- ***Addition***

$$\tilde{A}_1 \oplus \tilde{A}_2 = \Big\langle \tau_1 + \tau_2 - \tau_1\tau_2,\ \chi_1 + \chi_2 - \chi_1\chi_2,\ \iota_1\iota_2,\ \nu_1\nu_2,\ \phi_1\phi_2 \Big\rangle.$$

- ***Multiplication***

$$\tilde{A}_1 \otimes \tilde{A}_2 = \Big\langle \tau_1\tau_2,\ \chi_1\chi_2,\ \iota_1 + \iota_2 - \iota_1\iota_2,\ \nu_1 + \nu_2 - \nu_1\nu_2,\ \phi_1 + \phi_2 - \phi_1\phi_2 \Big\rangle.$$

- ***Scalar Multiplication***

$$k * \tilde{A}_1 = \Big\langle 1 - (1 - \tau_1)^k,\ 1 - (1 - \chi_1)^k,\ (\iota_1)^k,\ (\nu_1)^k,\ (\phi_1)^k \Big\rangle.$$

where $k \in [0, 1]$.

- ***Power Operation***

$$(\tilde{A}_1)^\alpha = \Big\langle (\tau_1)^\alpha,\ (\chi_1)^\alpha,\ 1 - (1 - \iota_1)^\alpha,\ 1 - (1 - \nu_1)^\alpha,\ 1 - (1 - \phi_1)^\alpha \Big\rangle.$$

where $\alpha \in [0, 1]$.

Definition 5. (**Normalization of PNN**) [17] *For a PNN as* $\tilde{A} = \langle \tau, \chi, \iota, \nu, \phi \rangle$, *normalization is performed with respect to the nature of criteria as follows:*

$$(i)\ \textbf{\textit{Benefit criteria}} = (\tilde{A})' = \langle \tau, \chi, \iota, \nu, \phi \rangle.$$
$$(ii)\ \textbf{\textit{Cost criteria}} = (\tilde{A})' = \langle \phi, \nu, 1 - \iota, \chi, \tau \rangle. \tag{1}$$

$(\tilde{A})'$ *is normalized PNN of* $\tilde{A}$.

Definition 6. (**Score Function**) *The score function* [22] *of a PNN* $\tilde{A} = \langle \tau, \chi, \iota, \nu, \phi \rangle$ *is defined as*

$$S(\tilde{A}) = \frac{(3 + \tau + \chi) - (\iota + \nu + \phi)}{5}. \tag{2}$$

It provides a crisp representation to rank alternatives under PNN environment. The above score function is bounded in [0,1].

Definition 7. (**Accuracy Function**) *The accuracy function* [22] *of a PNN* $\tilde{A} = \langle \tau, \chi, \iota, \nu, \phi \rangle$ *is defined as*

$$Acc(\tilde{A}) = \tau + \chi - \nu - \phi$$

The accuracy function also produces a crisp value of PNN.
The accuracy function is bounded in [-1,1].

Definition 8. (**Ranking rules**) [22] *Let* $\tilde{A}, \tilde{B}$ *be two PNN, then*
If $S(\tilde{A}) > S(\tilde{B})$ *then* $\tilde{A} > \tilde{B}$
If $S(\tilde{A}) = S(\tilde{B})$ *but* $Acc(\tilde{A}) > Acc(\tilde{B})$ *then* $\tilde{A} > \tilde{B}$
If $S(\tilde{A}) = S(\tilde{B})$ *and* $Acc(\tilde{A}) = Acc(\tilde{B})$ *then* $\tilde{A} = \tilde{B}$

Definition 9. (**Aggregation Operators on PNN**)

- *The Pentapartitioned Neutrosophic Weighted Arithmetic Aggregation (PN-WAA) operator* [20], *is defined as*

$$PNWAA(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m) = \oplus_{i=1}^{m} w_i * \tilde{A}_i =$$
$$\Big\langle 1-\prod_{i=1}^{m}(1-\tau_i)^{w_i},\ 1-\prod_{i=1}^{m}(1-\chi_i)^{w_i},\ \prod_{i=1}^{m}(\iota_i)^{w_i},\ \prod_{i=1}^{m}(\nu_i)^{w_i},\ \prod_{i=1}^{m}(\phi_i)^{w_i}\Big\rangle. \tag{3}$$

- *The Pentapartitioned Neutrosophic Weighted Geometric Aggregation (PN-WGA) operator* [17], *is defined as*

$$PNWGA(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m) = \otimes_{i=1}^{n} \tilde{A}_i^{w_i} =$$
$$\Big\langle \prod_{i=1}^{m}(\tau_i)^{w_i},\ \prod_{i=1}^{m}(\chi_i)^{w_i},\ 1-\prod_{i=1}^{m}(1-\iota_i)^{w_i},\ 1-\prod_{i=1}^{m}(1-\nu_i)^{w_i},\ 1-\prod_{i=1}^{m}(1-\phi_i)^{w_i}\Big\rangle. \tag{4}$$

  Where, $W = [w_1, w_2, ..., w_m]^T$ *represents the weight vector of the PNNs* $\tilde{A}_i = \langle \tau_i, \chi_i, \iota_i, \nu_i, \phi_i \rangle$, $i = 1, 2..., m$ *respectively, such that* $\sum_{i=1}^{m} w_i = 1$ *and* $0 \leq w_i \leq 1$.

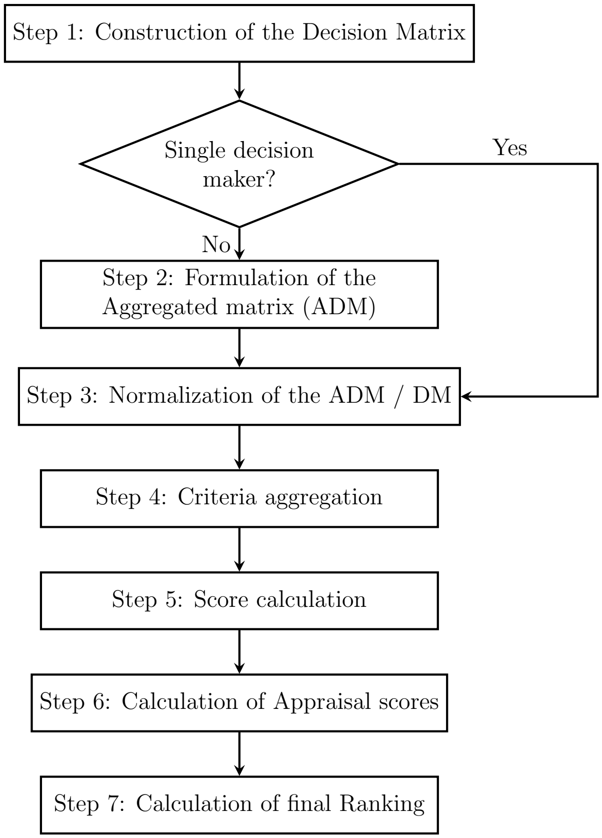
Definition 10. (Decision Matrix) *The decision matrix with* i *Alternatives denoted by* $AT_1, AT_2, \cdots, AT_i$ *and* j *Criteria denoted by* $CR_1, CR_2, \cdots, CR_j$ *is a* $i \times j$ *matrix given as* $A = [a_{ij}]_{i\times j}$ *where every element of this matrix is a PNN as*

$$a_{ij} = \langle \tau_{ij}, \chi_{ij}, \iota_{ij}, \nu_{ij}, \phi_{ij} \rangle. \tag{5}$$

3. PNN-COCOSO method for decision making

Consider a group decision making problem with i alternatives and j criteria involving n Decision Makers(DMs).
Alternatives : $AT_1, AT_2, AT_3, ..., AT_i$.
Criteria : $CR_1, CR_2, CR_3, ..., CR_j$.
Decision Makers (DMs) : $DM_1, DM_2, DM_3, ..., DM_n$.
Criteria weight vector : $W' = [w'_1, w'_2, ..., w'_j]^T$ such that $0 \leq w'_j \leq 1$ and $\sum_{i=1}^{j} w'_i = 1$.
DM weight vector : $W = [w_1, w_2, ..., w_n]^T$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$.

The Multi-Attribute Group Decision Making (MAGDM) process is depicted in the flowchart below and described in the following steps. The flowchart also addresses the case of a single DM (MADM).



- **Step 1: Construction of the decision matrices**
  Construct n decision matrices with respect to the ratings provided by n different decision makers in PNN format as $A^{(1)}, A^{(2)}, ..., A^{(n)}$. The decision making matrix for the $k-th$ decision maker is denoted by $A^{(k)}$ and defined as,

$$A^{(k)} = \begin{array}{c|ccc} & CR_1 & CR_2 & \cdots CR_j \\ \hline AT_1 & \langle \tau_{11}^k, \chi_{11}^k, \iota_{11}^k, \nu_{11}^k, \phi_{11}^k \rangle & \cdots & \langle \tau_{1j}^k, \chi_{1j}^k, \iota_{1j}^k, \nu_{1j}^k, \phi_{1j}^k \rangle \\ AT_2 & \langle \tau_{21}^k, \chi_{21}^k, \iota_{21}^k, \nu_{21}^k, \phi_{21}^k \rangle & \cdots & \langle \tau_{2j}^k, \chi_{2j}^k, \iota_{2j}^k, \nu_{2j}^k, \phi_{2j}^k \rangle \\ \vdots & \ddots & \vdots & \\ AT_i & \langle \tau_{i1}^k, \chi_{i1}^k, \iota_{i1}^k, \nu_{i1}^k, \phi_{i1}^k \rangle & \cdots & \langle \tau_{ij}^k, \chi_{ij}^k, \iota_{ij}^k, \nu_{ij}^k, \phi_{ij}^k \rangle. \end{array} \quad (6)$$

- **Step 2: Formulation of the Aggregated Decision Matrix (ADM)**
  By using PNWAA operator (see Eq. (3)), all the decision matrices are combined to formulate the ADM, denoted as $\mathcal{A} = [\delta_{ij}]_{i\times j}$, whose entries are generated by,

$$\delta_{ij} = \left\langle 1-\prod_{k=1}^{n}(1-\tau_{ij}^{k})^{w_k},\ 1-\prod_{k=1}^{n}(1-\chi_{ij}^{k})^{w_k},\ \prod_{k=1}^{n}(\iota_{ij}^{k})^{w_k},\ \prod_{k=1}^{n}(\nu_{ij}^{k})^{w_k},\ \prod_{k=1}^{n}(\phi_{ij}^{k})^{w_k} \right\rangle, \tag{7}$$

  where $w_1, w_2, ..., w_n$ denote the DM weights.

  Using Eq. (7), we establish the ADM ($\mathcal{A}$) as

$$\mathcal{A} = \begin{array}{c|cccc} & CR_1 & CR_2 & \cdots & CR_j \\ \hline AT_1 & \delta_{11} & \delta_{12} & \cdots & \delta_{1j} \\ AT_2 & \delta_{21} & \delta_{22} & \cdots & \delta_{2j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ AT_i & \delta_{i1} & \delta_{i2} & \cdots & \delta_{ij} \end{array} \tag{8}$$

  .

  Note: In the case of a single decision maker (MADM) the aggregation of decision matrices are avoided and we move to next step with the same initial decision matrix.

- **Step 3: Normalization of the ADM**
  We normalize the ADM($\mathcal{A}$) in Eq. (8) using Eq. (1) and establish a Normalized ADM (NADM) matrix $\mathcal{B}$ as

$$\mathcal{B} = [\sigma_{ij}]_{i\times j}, \qquad \text{where we assume} \qquad \sigma_{ij} = \langle p_{ij}, q_{ij}, r_{ij}, s_{ij}, t_{ij} \rangle. \tag{9}$$

  To satisfy the fundamental MADM requirement for criteria homogeneity, this normalization standardizes varying dimensions into a uniform scale.

- **Step 4: Criteria aggregation**
  To generate a single, consolidated PNN for each alternative, we aggregate the criteria from the NADM (Eq. 9) based on their respective weights. This aggregation is computed in two parallel steps to form two $i \times 1$ column matrices:

  - **Additive Aggregation Matrix (AAM),** $\mathcal{P} = [\vartheta_i]_{i\times 1}$: Derived using the PNWAA operator (Eq. 3), with its elements ϑ_i computed from Eq. (10).

- **Multiplicative Aggregation Matrix (MAM),** $\mathcal{Q} = [\upsilon_i]_{i\times 1}$: Derived using the PNWGA operator (Eq. 4), with its elements υ_i computed from Eq. (11).

$$\vartheta_i = \left\langle 1-\prod_{k=1}^{j}(1-p_{ik})^{w'_k},\ 1-\prod_{k=1}^{j}(1-q_{ik})^{w'_k},\ \prod_{k=1}^{j}(r_{ik})^{w'_k},\ \prod_{k=1}^{j}(s_{ik})^{w'_k},\ \prod_{k=1}^{j}(t_{ik})^{w'_k} \right\rangle, \tag{10}$$

$$\upsilon_i = \left\langle \prod_{k=1}^{j}(p_{ik})^{w'_k},\ \prod_{k=1}^{j}(q_{ik})^{w'_k},\ 1-\prod_{k=1}^{j}(1-r_{ik})^{w'_k},\ 1-\prod_{k=1}^{j}(1-s_{ik})^{w'_k},\ 1-\prod_{k=1}^{j}(1-t_{ik})^{w'_k} \right\rangle, \tag{11}$$

where $W' = [w'_1, w'_2, \ldots, w'_j]^T$ denotes the criteria weight vector. This defines the AAM (Eq. (12)) and MAM (Eq. (13)) matrices as presented below.

$$\mathcal{P} = \begin{array}{c|c} & \text{AAM} \\ \hline AT_1 & \vartheta_1 \\ AT_2 & \vartheta_2 \\ \vdots & \vdots \\ AT_i & \vartheta_i \end{array} \tag{12}$$

$$\mathcal{Q} = \begin{array}{c|c} & \text{MAM} \\ \hline AT_1 & \upsilon_1 \\ AT_2 & \upsilon_2 \\ \vdots & \vdots \\ AT_i & \upsilon_i \end{array} \tag{13}$$

- **Step 5: Score calculation**
  Using the score function from Eq. (2) we calculate the score (crisp value) for every PNN corresponding to every alternative for the matrices AAM($\mathcal{P}$) in Eq. (12) and MAM($\mathcal{Q}$) in Eq. (13) , which are denoted as , $Score(\mathcal{P}) = \mathcal{P}_i$ and $Score(\mathcal{Q}) = \mathcal{Q}_i$ respectively, where $i = 1, 2, 3....n$ denotes the alternatives.

- **Step 6: Calculation of Appraisal scores**
  Now, with respect to the obtained score values, we calculate three appraisal scores for every alternative as K_{ia}, K_{ib}, K_{ic} [29] using the Eqs. (14)-(16).

$$K_{ia} = \frac{(\mathcal{Q}_i) + (\mathcal{P}_i)}{\sum_{i=1}^{n}((\mathcal{Q}_i) + (\mathcal{P}_i))} \tag{14}$$

$$K_{ib} = \frac{(\mathcal{P}_i)}{\min_i(\mathcal{P}_i)} + \frac{(\mathcal{Q}_i)}{\min_i(\mathcal{Q}_i)} \tag{15}$$

$$K_{ic} = \frac{\Omega(\mathcal{P}_i) + (1-\Omega)(\mathcal{Q}_i)}{\Omega \max_i(\mathcal{P}_i) + (1-\Omega)\max_i(\mathcal{Q}_i)}, \qquad 0 \le \Omega \le 1 \tag{16}$$

where $i = 1, 2....., n$ denotes the number of alternatives and Ω denotes the distinguished parameter.

Note - The distinguished parameter is inherent to the COCOSO method [29] which acts as a compromise measure to balance between the AAM and MAM, based on weighted sum and weighted product to produce more comprehensive ranking results.

- **Step 7: Calculation of final Ranking**
  Finally, we compute the value of K [29] to calculate the final score using the formula given below as,

$$K = \frac{K_{ia} + K_{ib} + K_{ic}}{3} + (K_{ia} K_{ib} K_{ic})^{1/3} \tag{17}$$

  Then we rank the alternatives based on the final score in descending order to identify the best one. The alternative with highest score value K is selected as the best one.

4. Illustration of the proposed methodology

To demonstrate the proposed methodology, this study employs a **benchmark-based** validation strategy by using pre-validated datasets from peer-reviewed literature. This approach satisfies the structural requirements of the PNN framework, providing a reliable baseline to compare the proposed COCOSO logic against existing results.

4.1. Illustration 1 - Group decision making (MAGDM)

The group decision making problem depicted here with all qualitative and quantitative inputs have been adopted from a published paper involving SVPNN-ARAS method [20].

Problem outline :

In the problem four potential suppliers are considered for evaluation which act as our alternatives ($AT_1,\ AT_2,\ AT_3,\ AT_4$), out of which the best is to be selected based on predefined criteria. Three decision makers (DM_1, DM_2, DM_3) are employed from different departments to provide ratings to the alternatives based on three selected criteria. The criteria are assigned weights based on their relative importance in decision making. Decision makers are also assigned weights based on their proficiency and expertise.

Alternatives :

$$AT_1,\ AT_2,\ AT_3,\ AT_4.$$

Criteria : CR_1(Product quality), CR_2(Pollution control efficiency), CR_3(Environment management).

Note: Since the criteria measure the product quality, pollution control efficiency and environmental management, they are considered to be of benefit type.

Decision Makers :DM_1 (Production department), DM_2 (Purchase department), DM_3 (Quality control department).

Criteria Weights :

$$W' = [0.28,\ 0.31,\ 0.41]^T, \qquad \text{hence} \qquad w'_1 = 0.28,\ w'_2 = 0.31,\ w'_3 = 0.41$$

Decision Maker Weights:

$$W = [0.25,\ 0.41,\ 0.34]^T, \qquad \text{hence} \qquad w_1 = 0.25,\ w_2 = 0.41,\ w_3 = 0.34$$

- **Step 1: Construction of Decision matrices**
  The decision matrices [20] for three decision makers is shown in Table 1, Table 2 and Table 3 below.

Table 1: Decision matrix ($A^{(1)}$) with PNN (for DM_1)

	CR_1	CR_2	CR_3
AT_1	$\langle 0.48, 0.32, 0.45, 0.21, 0.27\rangle$	$\langle 0.53, 0.52, 0.58, 0.56, 0.34\rangle$	$\langle 0.74, 0.75, 0.56, 0.45, 0.43\rangle$
AT_2	$\langle 0.74, 0.42, 0.42, 0.37, 0.28\rangle$	$\langle 0.45, 0.22, 0.71, 0.58, 0.29\rangle$	$\langle 0.54, 0.55, 0.66, 0.38, 0.31\rangle$
AT_3	$\langle 0.71, 0.53, 0.70, 0.65, 0.75\rangle$	$\langle 0.53, 0.45, 0.75, 0.58, 0.69\rangle$	$\langle 0.74, 0.52, 0.62, 0.32, 0.48\rangle$
AT_4	$\langle 0.51, 0.57, 0.64, 0.62, 0.48\rangle$	$\langle 0.62, 0.45, 0.42, 0.81, 0.55\rangle$	$\langle 0.87, 0.42, 0.24, 0.59, 0.34\rangle$

Table 2: Decision matrix ($A^{(2)}$) with PNN for (DM_2)

	CR_1	CR_2	CR_3
AT_1	$\langle 0.68, 0.62, 0.35, 0.51, 0.36\rangle$	$\langle 0.35, 0.72, 0.52, 0.62, 0.34\rangle$	$\langle 0.64, 0.82, 0.92, 0.55, 0.43\rangle$
AT_2	$\langle 0.64, 0.38, 0.58, 0.75, 0.48\rangle$	$\langle 0.85, 0.52, 0.82, 0.42, 0.39\rangle$	$\langle 0.74, 0.55, 0.85, 0.23, 0.41\rangle$
AT_3	$\langle 0.61, 0.87, 0.28, 0.47, 0.35\rangle$	$\langle 0.36, 0.71, 0.22, 0.58, 0.50\rangle$	$\langle 0.74, 0.55, 0.64, 0.42, 0.26\rangle$
AT_4	$\langle 0.22, 0.62, 0.44, 0.79, 0.25\rangle$	$\langle 0.82, 0.46, 0.78, 0.40, 0.32\rangle$	$\langle 0.65, 0.56, 0.34, 0.23, 0.34\rangle$

Table 3: Decision matrix ($A^{(3)}$) with PNN for (DM_3)

	CR_1	CR_2	CR_3
AT_1	$\langle 0.48, 0.65, 0.45, 0.31, 0.47\rangle$	$\langle 0.53, 0.62, 0.43, 0.30, 0.32\rangle$	$\langle 0.64, 0.82, 0.42, 0.25, 0.53\rangle$
AT_2	$\langle 0.84, 0.26, 0.45, 0.37, 0.38\rangle$	$\langle 0.85, 0.52, 0.21, 0.50, 0.39\rangle$	$\langle 0.64, 0.35, 0.56, 0.48, 0.31\rangle$
AT_3	$\langle 0.81, 0.72, 0.36, 0.27, 0.35\rangle$	$\langle 0.53, 0.65, 0.33, 0.40, 0.29\rangle$	$\langle 0.64, 0.427, 0.321, 0.32, 0.38\rangle$
AT_4	$\langle 0.41, 0.77, 0.24, 0.52, 0.22\rangle$	$\langle 0.92, 0.75, 0.52, 0.39, 0.25\rangle$	$\langle 0.77, 0.725, 0.458, 0.39, 0.34\rangle$

- **Step 2: Formation of Aggregated Decision Matrix (ADM)**

  The ADM ($\mathcal{A}$) obtained using the Eq. (7) is displayed in Table 4.

Table 4: ADM ($\mathcal{A}$) with PNN

	CR_1	CR_2	CR_3
AT_1	$\langle 0.574, 0.572, 0.406, 0.345, 0.367\rangle$	$\langle 0.463, 0.644, 0.501, 0.472, 0.333\rangle$	$\langle 0.668, 0.805, 0.622, 0.400, 0.462\rangle$
AT_2	$\langle 0.748, 0.352, 0.490, 0.494, 0.387\rangle$	$\langle 0.792, 0.458, 0.498, 0.483, 0.362\rangle$	$\langle 0.665, 0.490, 0.664, 0.335, 0.348\rangle$
AT_3	$\langle 0.716, 0.767, 0.383, 0.422, 0.423\rangle$	$\langle 0.466, 0.637, 0.343, 0.511, 0.450\rangle$	$\langle 0.709, 0.507, 0.475, 0.358, 0.345\rangle$
AT_4	$\langle 0.368, 0.669, 0.393, 0.645, 0.282\rangle$	$\langle 0.835, 0.582, 0.582, 0.473, 0.337\rangle$	$\langle 0.763, 0.599, 0.351, 0.348, 0.340\rangle$

- **Step 3: Normalization of ADM**

  Here normalization is not required as all the criteria are of benefit type. So the normalized ADM ($\mathcal{B}$) is same as above which is represented in Table 4.

- **Steps 4 and 5: Criteria aggregation with score calculation**

  The AAM ($\mathcal{P}$ matrix) (see Eq. (10)) and MAM ($\mathcal{Q}$ matrix) (see Eq. (11)) with corresponding score values (Eq. (2)) are displayed in Table 5 and Table 6 respectively.

Table 5: AAM ($\mathcal{P}$) with Scores

	PNN Values combining CR_1, CR_2, CR_3	Scores
AT_1	$\langle 0.58684, 0.70714, 0.51628, 0.40404, 0.39121\rangle$	0.67473009
AT_2	$\langle 0.7337, 0.44440, 0.55816, 0.41838, 0.36294\rangle$	0.64024639
AT_3	$\langle 0.65167, 0.63661, 0.40468, 0.41855, 0.39674\rangle$	0.69301148
AT_4	$\langle 0.72149, 0.61539, 0.42388, 0.45511, 0.32166\rangle$	0.69157728

Table 6: MAM ($\mathcal{Q}$) with Scores

	PNN Values combining CR_1, CR_2, CR_3	Scores
AT_1	$\langle 0.57153, 0.68290, 0.53264, 0.40907, 0.39796\rangle$	0.582952818
AT_2	$\langle 0.72570, 0.43760, 0.57269, 0.43032, 0.36350\rangle$	0.559357243
AT_3	$\langle 0.71458, 0.61103, 0.41161, 42706, 0.40134\rangle$	0.617120896
AT_4	$\langle 0.42804, 0.61266, 0.44437, 0.48530, 0.32320\rangle$	0.55756451

- **Steps 6 and 7: Calculation of Appraisal scores with final ranking**

  Using the Eqs. (14)- (16), we calculate the appraisal scores (K_{ia} , K_{ib} and K_{ic}) of each alternative. The **FINAL** score (K) is calculated using Eq.(17), considering the value of the distinguished parameter $\Omega = 0.5$. The values of all the scores are reflected in the Table 7. Hence, observing the value of K

Table 7: Computed values of K_{ia}, K_{ib}, K_{ic}, and K for alternatives

	K_{ia}	K_{ib}	K_{ic}	K
AT_1	0.250706	2.099394	0.959966	1.901644
AT_2	0.239129	2.003215	0.915635	1.814541
AT_3	0.261161	2.189229	1.000000	1.981657
AT_4	0.249004	2.080174	0.953447	1.886506

we can determine the ranking as

$$AT_3 > AT_1 > AT_4 > AT_2$$

which shows A_3 is the best alternative and A_2 is the worst

- **Sensitivity Analysis**
  For different values of distinguished parameter $\Omega = 0.1, 0.2, ..., 1.0$ (Eqs. (16)-(17)), the rankings obtained are shown in the Table 8. Here we can see, based

Table 8: Ranking of alternatives based on parameter Ω

Ω	AT_1	AT_2	AT_3	AT_4	Ranking
0.1	1.8942	1.8101	1.9816	1.8624	$AT_3 > AT_1 > AT_4 > AT_2$
0.2	1.8962	1.8112	1.9816	1.8686	$AT_3 > AT_1 > AT_4 > AT_2$
0.3	1.8980	1.8124	1.9816	1.8748	$AT_3 > AT_1 > AT_4 > AT_2$
0.4	1.8999	1.8135	1.9816	1.8807	$AT_3 > AT_1 > AT_4 > AT_2$
0.5	1.9016	1.8145	1.9816	1.8865	$AT_3 > AT_1 > AT_4 > AT_2$
0.6	1.9034	1.8155	1.9816	1.8921	$AT_3 > AT_1 > AT_4 > AT_2$
0.7	1.9051	1.8166	1.9816	1.8976	$AT_3 > AT_1 > AT_4 > AT_2$
0.8	1.9067	1.8176	1.9816	1.9030	$AT_3 > AT_1 > AT_4 > AT_2$
0.9	1.9083	1.8185	1.9816	1.9082	$AT_3 > AT_1 > AT_4 > AT_2$
1.0	1.9099	1.8195	1.9816	1.9133	$AT_3 > AT_1 > AT_4 > AT_2$

on the computed values of K, the alternatives always rank in the same order as

$$AT_3 > AT_1 > AT_4 > AT_2$$

for all possible values of the distinguished parameter Ω. The distribution of scores with ranking are also shown in the Figure-1.

Note- All values are rounded up to 4 decimal places for presentation purpose.

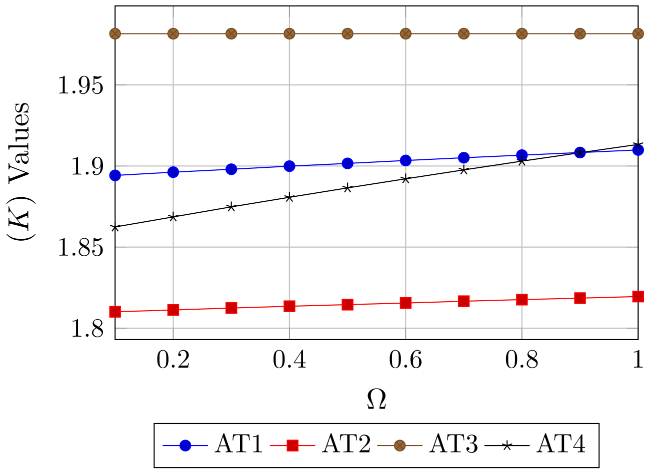


Figure 1: Variation of (K) values with respect to Ω

4.2. Illustration 2- Single decision maker (MADM)

Fuel injector problem [22]

The problem presented here involves a single decision maker and has been adopted from a published paper of Dombi-operators in PNN environment. [22]

Problem outline :

In the problem, three detergents are considered for evaluation of their cleaning efficiency on fuel injectors, which act as our alternatives. The evaluation is rated against four performance criteria.

Alternatives :

$$AT_1,\ AT_2,\ AT_3.$$

Criteria :

$$CR_1,\ CR_2,\ CR_3,\ CR_4.$$

Note- Since the criteria defined above measure the performance efficiency they are considered to be of benefit type. The criteria are assigned weights based on their relative importance in decision making.

Criteria Weights :

$$W' = [0.27,\ 0.2,\ 0.3,\ 0.33]^T, \quad \text{hence} \quad w'_1 = 0.27,\ w'_2 = 0.2,\ w'_3 = 0.3,\ w'_4 = 0.33$$

- **Step 1: Construction of Decision matrices**
  The decision matrices [22] for three decision makers is shown in Table 9 below.

Table 9: Decision matrix with PNN values

	CR$_1$	CR$_2$	CR$_3$	CR$_4$
AT$_1$	$\langle 0.5, 0.6, 0.8, 0.2, 0.1\rangle$	$\langle 0.6, 0.5, 0.6, 0.6, 0.1\rangle$	$\langle 0.3, 0.4, 0.5, 0.6, 0.3\rangle$	$\langle 0.6, 0.6, 0.5, 0.3, 0.1\rangle$
AT$_2$	$\langle 0.5, 0.6, 0.4, 0.3, 0.3\rangle$	$\langle 0.6, 0.5, 0.8, 0.4, 0.6\rangle$	$\langle 0.4, 0.5, 0.3, 0.4, 0.4\rangle$	$\langle 0.3, 0.4, 0.5, 0.4, 0.2\rangle$
AT$_3$	$\langle 0.7, 0.6, 0.5, 0.4, 0.4\rangle$	$\langle 0.7, 0.5, 0.4, 0.6, 0.7\rangle$	$\langle 0.7, 0.9, 0.4, 0.4, 0.5\rangle$	$\langle 0.6, 0.4, 0.5, 0.6, 0.6\rangle$

- **Steps 2 and 3: Formation of Aggregated Decision Matrix (ADM) and normalization** Since a single decision maker is employed to rate the alternatives, aggregation is not required here. Moreover, since all the criteria are of benefit type, normalization is also not required. Hence, the normalized DM is same as above which is represented in Table 9.

- **Steps 4 and 5: Criteria aggregation with score calculation**
  The AAM ($\mathcal{P}$ matrix) (see Eq. (10)) and MAM ($\mathcal{Q}$ matrix) (see Eq. (11)) with corresponding score values (Eq. (2)) are displayed in Table 10 and Table 11 respectively.

Table 10: AAM ($\mathcal{P}$) with Scores

	PNN Values combining CR_1, CR_2, CR_3, CR_4	Scores
AT$_1$	$\langle 0.5415, 0.5690, 0.5493, 0.3371, 0.1104\rangle$	0.6448
AT$_2$	$\langle 0.4734, 0.5335, 0.4140, 0.3377, 0.2913\rangle$	0.6510
AT$_3$	$\langle 0.7075, 0.7122, 0.4173, 0.4525, 0.4989\rangle$	0.7100

Table 11: MAM ($\mathcal{Q}$) with Scores

	PNN Values combining CR_1, CR_2, CR_3, CR_4	Scores
AT$_1$	$\langle 0.4408, 0.4867, 0.5493, 0.5998, 0.6\rangle$	0.4357
AT$_2$	$\langle 0.3823, 0.4552, 0.4140, 0.4801, 0.6186\rangle$	0.4650
AT$_3$	$\langle 0.6420, 0.5430, 0.4173, 4743, 0.5301\rangle$	0.5527

- **Steps 6 and 7: Calculation of Appraisal scores with final Ranking K:**
  The appraisal scores of each alternative, denoted by K_{ia}, K_{ib}, and K_{ic}, are calculated using Eqs. (14)-(16). The final score (K) is generated using Eq. (17), considering the value of the distinguished parameter $\Omega = 0.5$. The values of all scores and the final ranking are presented in Table 12. Hence,

Table 12: Computed values of K_{ia}, K_{ib}, K_{ic}, and K for alternatives

	K_{ia}	K_{ib}	K_{ic}	K
AT_1	0.3123	2.0	0.8557	1.8693
AT_2	0.3226	2.0770	0.8839	1.9357
AT_3	0.3650	2.3696	1.0000	2.1981

observing the value of K we can determine the ranking as

$$AT_3 > AT_2 > AT_1$$

which shows A_3 is the best alternative and A_1 is the worst.

- **Sensitivity Analysis**
  For different values of distinguished parameter $\Omega = 0.1, 0.2, ..., 1.0$ (Eqs. (16)-(17)), the rankings are shown in the Table 13.

Table 13: Ranking of alternatives based on parameter Ω

Ω	AT_1	AT_2	AT_3	Ranking
0.1	1.8350	1.9142	2.1981	$AT_3 > AT_2 > AT_1$
0.2	1.8444	1.9200	2.1981	$AT_3 > AT_2 > AT_1$
0.3	1.8532	1.9255	2.1981	$AT_3 > AT_2 > AT_1$
0.4	1.8615	1.9308	2.1981	$AT_3 > AT_2 > AT_1$
0.5	1.8693	1.9357	2.1981	$AT_3 > AT_2 > AT_1$
0.6	1.8767	1.9404	2.1981	$AT_3 > AT_2 > AT_1$
0.7	1.8838	1.9449	2.1981	$AT_3 > AT_2 > AT_1$
0.8	1.8905	1.9491	2.1981	$AT_3 > AT_2 > AT_1$
0.9	1.8969	1.9532	2.1981	$AT_3 > AT_2 > AT_1$
1.0	1.9029	1.9570	2.1981	$AT_3 > AT_2 > AT_1$

Here we can see, based on the computed values of K, the alternatives always rank in the same order as

$$AT_3 > AT_2 > AT_1$$

for all possible values of the distinguished parameter $\Omega \in (0, 1]$.

5. Results and Discussions

By applying the PNN-COCOSO method to the Green supplier problem and Fuel injector problem as depicted above, we obtained the final ranking as

Green supplier problem

$$AT_3 > AT_1 > AT_4 > AT_2$$

Fuel injector problem

$$AT_3 > AT_1 > AT_2$$

for the value of the distinguished parameter $\Omega = 0.5$. To investigate the parametric sensitivity of the model, we also vary the distinguished parameter Ω in (Eq. 16) from $\Omega = 0.1$ to $\Omega = 1.0$, to verify alterations in ranking. The results reveal the robustness and stability of the method, showing no alteration of ranking throughout the process. It also validates the proposed method in both scenarios considering single and group decision making.

6. Comparative Analysis

To validate the robustness and supremacy of the proposed PNN-COCOSO method, a comprehensive comparative analysis is conducted from theoretical and empirical points of view.

6.1. Theoretical analysis

The proposed PNN-COCOSO method demonstrates theoretical superiority over existing fuzzy, neutrosophic, and PNN-based MADM frameworks. As discussed earlier, compared to complex neutrosophic models (Aczel-Alsina [11] or Prioritized Muirhead Mean [12]), PNN-COCOSO provides a more precise measurement of uncertainty through its granular five-component structure. Moreover, while supply chain fuzzy hybrid models, such as SWARA-MABAC [18], LODECI-CORASO [27], and Fuzzy DEMATEL/BWM combinations [10], primarily focus on criteria and sub-criteria selection, the proposed framework utilizes multiple aggregation and scoring methods to stabilize rankings while effectively handling uncertainty. Additionally, most of these existing hybrid models frequently fail to synthesize group opinions and are limited to single decision-makers, whereas PNN-COCOSO accommodates both single and group decision-makers simultaneously under a unified algorithmic procedure. Furthermore, unlike the SVPNN-ARAS [20] strategy's reliance on a single utility ratio relative to an optimal value, PNN-COCOSO employs three distinct appraisal scores (K_{ia}, K_{ib}, K_{ic}) alongside integrated additive and multiplicative aggregations. Finally, in contrast to Dombi-operator methods [22], which also operate within the PNN environment but lack combined aggregation, depend heavily on operational parameters, and are devoid of a conventional MADM framework, the PNN-COCOSO strategy successfully addresses all these structural deficiencies.

6.2. Empirical analysis

6.2.1. Comparison with SVNS-ARAS strategy [20]

In the original study, the SVNS-ARAS method yielded a final ranking of $AT_2 > AT_3 > AT_1 > AT_4$. However, when processing the exact same decision matrices through the PNN-COCOSO framework, the ranking shifted to $AT_3 > AT_1 > AT_4 > AT_2$, where AT_3 emerges as the most viable alternative. This alteration can be attributed to the following factors:

- Difference in the score functions- ARAS uses a different score function which is not normalized to [0,1], in comparison PNN-COCOSO uses a normalized score function.
- ARAS uses a single aggregation procedure compared to additive and multiplicative aggregation in PNN-COCOSO which produces more balanced results.
- ARAS mainly relies on optimal value solution compared to multiple compromise measures in PNN-COCOSO method.

6.2.2. Comparison with Dombi-operator strategy [22] In the original study, both the PNDWAA (arithmetic) and PNDWGA (geometric) operators produced a consistent ranking of $A_3 > A_1 > A_2$ across varying operational parameters.

- The same ranking is also retained during the PNN-COCOSO model also following the same score function but it is consolidated through multiple compromise measures.
- PNN-COCOSO remains superior by combining both aggregation simultaneously to provide an output whereas in Dombi-operators the aggregation strategy has to be operated separately.
- Dombi-operator methods mainly focus on operator aggregation and is devoid of any conventional MADM framework whereas PNN-COCOSO provides both aggregation combined with an MADM strategy, claiming to be more beneficial.

7. Conclusion

This paper successfully demonstrates a novel PNN-COCOSO strategy for multi-attribute group decision making within the PNN environment. The methodology is meticulously designed to handle indeterminacy, hesitancy, and inconsistencies in decision-making by decomposing the indeterminacy component into contradiction, ignorance, and unknown elements through PNN frameworks. Although there exist several decision-making frameworks and aggregation operators, most are tailored strictly to fuzzy and neutrosophic settings and are inherently deficient in handling extreme multidimensional uncertainties. The proposed approach mitigates this by utilizing multiple aggregation operations, balanced by a distinguished parameter, to generate a highly consistent ranking.

To validate the operational framework and comparative superiority of the proposed method, two distinct problems from existing literature were analyzed.

Green supplier problem - Group decision-making

While the existing literature portrays a ranking of the four alternatives (suppliers) as $AT_2 > AT_3 > AT_1 > AT_4$, the application of the PNN-COCOSO method altered the ranking to $AT_3 > AT_1 > AT_4 > AT_2$, projecting AT_3 as the most viable alternative. This alteration in ranking highlights the vulnerabilities of the existing literature, which relies on a single aggregation technique and non-normalized score functions, whereas the proposed method captures a more accurate compromise solution through multiple aggregation and use of normalized score function.

Fuel injector problem - Single decision-maker

The proposed method produced a ranking of $AT_3 > AT_2 > AT_1$, which perfectly aligns with the existing literature. However, the proposed method establishes its superiority by leveraging the multiple aggregation strategies (weighted sum and weighted product) inherent to the COCOSO method, effectively addressing the literature's lack of a formalized and consistent MADM approach.

Both illustrations underwent a rigorous sensitivity analysis by comparing the results obtained through the variation of the distinguished parameter $\Omega \in [0, 1]$. The results revealed no alteration in the final rankings, confirming the high stability and robustness of the proposed methodology in highly uncertain environments.

This also brings to consideration a potential area of research in the future where more established MADM methods such as TOPSIS, VIKOR, and MABAC can be extended into the PNN environment using the foundational framework established here. Additionally, this study puts forward several compelling research questions regarding the integration of entropy measures for the objective determination of criteria weights to further enhance ranking consistency.

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